Counterfactuals and Possibilities

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A split in modal semantics

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- Nonclassical approaches (and especially an informational view) are becoming prevalent for epistemic modality.

- Nonclassical/informational approaches are still in the minority for other kinds of modality (counterfactual, circumstantial, some deontics).

- Even for existing nonclassical approaches to non-epistemic modality: the arguments for nonclassicality are very different.

This misses the generality of the classical truth-conditional view (Kratzer 2012).
A split in modal semantics

The main argument for nonclassical treatments of epistemic modals: nonclassical inferences.

**Epistemic Contradiction.** \( \neg A \land \Diamond A \not\models \bot \)

(1) #It's not raining and it might be raining.

- This inference cannot be captured on classical accounts of modality, i.e. any account that produces a logic that is classical.
- The proposed solution: a semantics based on information states.
These arguments don’t seem to generalize to other flavors of modality, in particular counterfactuals.

(2) It’s not raining, but it might have been. ✓

- Perhaps there are other sources of nonclassicality for counterfactuals (e.g. Reverse Sobel sequences; von Fintel 2001, Gillies 2007).
- But even so, they give rise to theories with different architecture.
The project

The goal for today: argue that this apparent difference is illusory.

- (Some) arguments for nonclassical theories for epistemics are instances of more general arguments for nonclassical theories of modality.
- The focus: counterfactuals, and the classical Stalnaker-Lewis debate.


- Possibilities are ‘thicker’ entities than possible worlds.
- Humberstone introduced possibilities to do modal logic without assuming ‘fully determined’ states of affairs. So the use of possibility is linked, from the beginning, to indeterminacy.
- Structurally, this is analogous to information state semantics.
• A solution to a persisting problem about counterfactuals.
Payoffs

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- New options for assigning probability to modal claims in a way that might align probabilities of conditionals and conditional probabilities.
• A **solution to a persisting problem** about counterfactuals.
• A **unified semantics** for various flavors of modality.
• New options for assigning **probability** to modal claims in a way that might align probabilities of conditionals and conditional probabilities.
• A new way of thinking about **modal expressivism**.
Background
**Variably strict semantics**

Variably strict semantics is the dominant approach to counterfactuals.

- The key feature: a comparative closeness relation $\preceq_w$.
- This induces a preorder on the domain of pw’s. (I.e.: a ranking with ties.)
- The function of $\preceq_w$: single out a set of ‘maximally close’ worlds to $w$. ($w'$ is maximally close $\preceq_w$ to $w$ iff, for all $w''$, $w' \preceq_w w''$.)

The resulting meaning:

$$\llbracket A \Box \rightarrow C \rrbracket^{\preceq, w} = \text{true iff}$$

for every maximally close $\preceq, w$ A-world $w'$, $C$ is true at $w'$
A problem for variably strict semantics: the following principle seems valid.

**Conditional Excluded Middle.** \( \models (A \rightarrow C) \lor (A \rightarrow \neg C) \)

One strong argument for CEM:

**Scopelessness.**

Counterfactuals are semantically scopeless (i.e., their meaning is unaffected by differences in syntactic scope) with respect to a large number of operators.
Scopelessness holds wrt negation...

(3)  
a. It’s not true that, if Frida had taken the exam, she would have passed.  
b. If Frida had taken the exam, she would not have passed.

...items that build negation into lexical meanings...

(4)  
a. I doubt that, if Frida had taken the exam, she would have passed.  
b. I believe that, if Frida had taken the exam, she would have failed.

...and quantifiers.

(5)  
a. Exactly nine of John’s students would pass, if he ran the exam.  
b. If John ran the exam, exactly nine of his students would pass.
The problem: variably strict semantics is quantificational, hence it predicts that counterfactuals have scope interactions (like other quantifiers).

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- And in fact we do observe these scope interactions with other modals.
- For an example: let’s replace *would* with a deontic modal.

(6) a. It’s not true that, if Frida takes the exam, she has to pass [in order to graduate]. ☞
   b. If Frida takes the exam, she has to not pass [in order to graduate].
So, why not switch to a semantics that vindicates CEM (like Stalnaker 1968)?
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There is a tension between CEM and another plausible principle.

**Would-Might Contradiction (WMC).** \((A \rightarrow \neg C) \land (A \leftrightarrow C) \models \bot\)

(7) # If Maria had passed, Frida would not have passed; but, even if Maria had passed, Frida might have passed.

(8) # Suppose that, if Maria had passed, Frida would not have passed; and that, even if Maria had passed, Frida might have passed.
CEM and WMC together entail an unacceptable principle:

**Collapse.** \[ A \leftrightarrow C \models A \Box \rightarrow C \]
CEM and WMC together entail an unacceptable principle:

**Collapse.** \( A \leftrightarrow C \models A \square \rightarrow C \)

(On a classical notion of consequence.)
CEM and WMC together entail an unacceptable principle:

**Collapse.** \[ A \leftrightarrow C \vdash A \Box \rightarrow C \]

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*Caveat*: keep WMC distinct from Lewis’s Duality:

**WMC.** \[ (A \Box \rightarrow \neg C) \land (A \leftrightarrow C) \vdash \bot \]

**Duality.** \[ (A \Box \rightarrow C) \not\models \neg (A \leftrightarrow \neg C) \]
Homogeneity?

A solution in the semantics literature: homogeneity.

• We treat conditionals as universal quantifiers, with the added constraint that all or none of the antecedent worlds make the consequent true.
• This is the standard analysis for plural definites like *the girls* in (9).

(9) The girls passed.

This analysis won’t work; ask if you’re interested in the details.
Expressivism
Yalcin (2007) points out that \( \neg A \) and \( \Diamond A \) seem inconsistent.

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(10) \# Suppose that it’s not raining and it might be raining.
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Yalcin’s puzzle is triggered by two classically inconsistent principles:

**Epistemic Contradiction.** ¬A ∧ ⊢A ⊨ ⊥

**Nonfactivity of Epistemic Modality.** ⊢A ⊭ A
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**Nonfactivity of Epistemic Modality.** \( \Diamond A \not\models A \)

Yalcin’s solution (building on Veltman 1996): move to a nonclassical semantics, where the basic unit of evaluation is an information state (modeled as a set of epistemic possibilities).
Our puzzle is triggered by three classically inconsistent principles.

Conditional Excluded Middle. (CEM) \( \equiv (A \Box \rightarrow C) \lor (A \Box \rightarrow \neg C) \)

Would-Might Contradiction. (WMC) \((A \Box \rightarrow \neg C) \land (A \diamond \leftarrow C) \models \bot\)

Nonfactivity of Might-Conditionals. (NMC) \(A \diamond \leftarrow C \not\models A \Box \rightarrow C\)

This is really a generalization of Yalcin’s puzzle. (Hint: plug in ‘\(\top\)’ for \(A\).)
Hence:

- The classical debate about CEM/Duality really brings up a tension for classical notions of consequence for conditionals.
Generalizing Yalcin’s puzzle

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- The classical debate about CEM/Duality really brings up a tension for classical notions of consequence for conditionals.
- This encourages exploring a nonclassical solution.
- If you take anything from this talk: the problem raised by epistemic contradictions generalizes in full to other flavors of modality.
Closeness and indeterminacy
As usual, a preorder

The starting point: Lewis’s picture of comparative closeness.

- Two-place relation $\preceq_w$ that is transitive and strongly connected.
- This induces a preorder on the domain of pw’s. (I.e.: a ranking with ties.)
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Bookkeeping assumptions: (i) I’m neutral on the interpretation of closeness; (ii) I assume a finite domain of worlds.
Two kinds of indeterminacy

The departure from Lewis: introduce indeterminacy in the picture.
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I assume two kinds of indeterminacy.

- indeterminacy about facts (about which world is actual).
- indeterminacy about counterfacts.
Factual indeterminacy

An example of factual indeterminacy: indeterminacy about the future.

- Suppose coin flips are indeterministic and Frida will flip a coin tomorrow.
- There are two candidates for the actual world: a tails and a heads world.
Two kinds of indeterminacy

We represent this via a ranking with multiple worlds at the top.
The second kind of indeterminacy: indeterminacy about what would have been
the case, on a counterfactual supposition.

- This is *indeterminacy about counterfacts*.
- Ties in the preorder ‘further down’ represent this kind of indeterminacy.
Missed Coin Flip. At $t_1$, there are three open possibilities: Frida may flip a coin tomorrow at noon and it might land tails; she might flip a coin and it might land heads; or she might not flip a coin. The third possibility comes about. It is now a later time, $t_2$, and there has been no coinflip.

- There is indeterminacy about what would have happened, on the supposition that the coin had been tossed.
- This is modeled via the tie between $w_F$ and $w_T$. 
Indeterminacy about counterfactuals

Why treats this as a case of indeterminacy?

A consideration from symmetry.

- At \( t_1 \), the tie between \( w_F \) and \( w_T \) was due to indeterminacy.
- Via symmetry, at the later time \( t_2 \), we should still regard the tie between them as due to indeterminacy.
Possibilities
A nonstandard of modal semantics: Humberstone’s *possibility semantics*. (See also Holliday 2015, in progress.)

Our entrée into the realm of the possible, one might say, is through imagining and storytelling about how things might be, considering what might happen under hypothetical circumstances, . . . but each such introduction into that realm presents us . . . a region of logical space, and not with a point thereof . . .

Here we have a motivation for the pursuit of modal logic against a semantic background in which less determinate entities than possible worlds, things which I am inclined for want of a better word to call simply possibilities are what sentences . . . are true or false with respect to.
Humberstone’s possibilities can be visualized as *nodes in a branching diagram.* (Where the branches have no end.)

- At each node, sentences can be true, false, or indeterminate.
- But *once they are true or false, they stay true or false.*
Some analogies with supervaluations:

- Some possibilities ‘refine’ other possibilities.
- A is true at a possibility $p$ just in case it’s true at all refinements of $p$. 

Hence there are no worlds.

For today’s purposes:
- Retaining or rejecting punctiformity makes no difference.
- The main point: the ‘thicker’ notion of possibility.

For simplicity, I will use ‘puncts’, i.e. perfectly precise possibilities.

(Though my ‘puncts’ won’t be worlds.)
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Possibility semantics for counterfactuals
I take a possibility to be a Lewisian preorder.

*Example.* Every fact about the history of the world is settled, aside from the outcome of Frida’s coin toss.
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The actual possibility, in this scenario:
Refinements

A (full) refinement (precisification, path) of a possibility $p$ is a total order that agrees with $p$ on all the non-ties, and ‘makes decisions’ about the ties.
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Intuitively: refinements are fully determinate possibilities, immune from both factual and counterfactual indeterminacy.

Formally, refinements are sequences of worlds.

\[
\begin{array}{c}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{array}
\Rightarrow
\begin{array}{c}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{array}
\]
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- Intuitively: refinements are **fully determinate possibilities**, immune from both factual and counterfactual indeterminacy.
- Formally, refinements are sequences of worlds.
**Example:** Frida considered flipping a coin, but she did not do so in the end.

One possibility with several refinements:

```
    w_F
   /   \
  w_H   w_T
     \  /
      : : :
      : : :
      : : :
```
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Two parameters for the semantics:

- a possibility $p$;
- an refinement $r$. 

Two parameters for the semantics:

- a **possibility** $p$;
- a **refinement** $r$.

**Truth at a possibility** is defined as **truth at all refinements**.

- In general, we define recursively truth at a refinement.
- But sometimes we define directly truth at a possibility.
Nomodal sentences are evaluated at the ‘factual’ part of refinements.

(11) Maria didn’t flip the coin.
Nomodal sentences are evaluated at the ‘factual’ part of refinements.

(11) Maria didn’t flip the coin. ✓
Nomodal sentences are evaluated at the ‘factual’ part of refinements.

(11) Maria didn't flip the coin.

(12) The coin landed heads or tails.

(13) The coin wasn't flipped or it landed tails.
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(13) The coin wasn’t flipped or it landed tails.
Nomodal sentences are evaluated at the ‘factual’ part of refinements.

(13) The coin wasn’t flipped or it landed tails. ✓
Would-counterfactuals are not quantifiers.

Rather: they ‘pick out’ an antecedent-verifying situation.

- Since refinements are sequences of worlds, they can be naturally used to evaluate counterfactuals.
How this works:

- We ‘update’ the refinement with the antecedent.
- We evaluate the consequent at the updated refinement.

(14) If Maria had flipped the coin, it would have landed heads.
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- We ‘update’ the refinement with the antecedent.
- We evaluate the consequent at the updated refinement.

If Maria had flipped the coin, ...

(14) If Maria had flipped the coin, it would have landed heads. ✓
**Might-counterfactuals** search for a refinement that, updated with the antecedent, makes the prejacent of *might* true.

\[
W_F \quad W_H \quad W_T
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(15) If Maria had flipped the coin, it might have landed tails.
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**Might-counterfactuals** search for a refinement that, updated with the antecedent, makes the prejacent of *might* true.

(15) If Maria had flipped the coin, it might have landed tails. ✓
Semantics, formally

\[ [A \square \rightarrow C]^{p,r} \text{ is true iff } p, \text{ updated with } A, \text{ makes } C \text{ true} \]

\[ [A \square \rightarrow C]^{p,r} = \text{ true iff } [C]^{p,r[A]} = \text{ true} \]

\[ [A \lozenge \rightarrow C]^{p,r} \text{ is true iff there is a refinement } r' \text{ of } p \text{ such that } r', \text{ updated with } A, \text{ makes } C \text{ true} \]

\[ [A \lozenge \rightarrow C]^{p,r} = \text{ true iff for some } r' \text{ s.t. } r' \text{ refines } p, [C]^{p,r'[A]} = \text{ true} \]
Truth and consequence

Truth at a possibility.

A is true at $p$ iff $A$ is true at all refinements of $p$

$p \vdash A$ iff $\forall r \in p, \llbracket A \rrbracket^{p,r} = 1$
Truth at a possibility.

$A$ is true at $p$ iff $A$ is true at all refinements of $p$

$p \models A$ iff $\forall r \in p, [A]^{p,r} = 1$

Logical consequence is preservation of truth at a possibility.

Logical consequence.

$A_1, \ldots, A_n \models C$ iff, for all $p$ such that $p \models A_1, \ldots, p \models A_n, p \models C$. 
This notion of consequence validates all the inferences that create our puzzle.

**Fact 1.** $\vdash (A \square \rightarrow C) \lor (A \square \rightarrow \neg C)$

**Fact 2.** $(A \square \rightarrow \neg C) \land (A \lozenge \rightarrow C) \vdash \bot$

**Fact 3.** $A \lozenge \rightarrow C \not\vdash A \square \rightarrow C$
The solution, intuitively

• Every refinement makes true either $A \square \rightarrow C$ or $A \square \rightarrow \neg C$, hence $(A \square \rightarrow C) \lor (A \square \rightarrow \neg C)$ is true at all possibilities.

• But: $A \square \rightarrow \neg C$ and $A \lozenge \rightarrow C$ can’t be true together.
  • $A \square \rightarrow \neg C$ is true iff all refinements, updated with $A$, make true $\neg C$
  • $A \lozenge \rightarrow C$ is true iff some refinement, updated with $A$, make true $C$
Thicker possibilities may be useful also in other modal domains.

- In fact, they are: Veltman/Yalcin-style information state semantics is formally very similar.
- This apparatus may easily generalize to other kinds of modality too.
The general line of argument.

- A **split in recent treatments of modality**: much work on informational semantics for epistemic modals, but still plenty of support for traditional accounts of counterfactuals.
- But: puzzles that lead to nonclassical semantics for epistemic modals generalize—in fact, those puzzles have been known since the beginning of the literature on counterfactuals.
The account.

- A semantics based on a Humberstone-style notion of possibility: thicker possibilities, that leave some questions unsettled.
- A sentence is true at a possibility iff it is true at all the refinements.
- In this framework, it’s easy to define meanings for would and might that solve the Lewis/Stalnaker puzzle.
Expressivism about modality is often likened to normative/metanormative expressivism in the style of Blackburn and Gibbard.
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But if modal expressivism is what I described, it is very different.

- Gibbard/Blackburn expressivism is motivated by the open question argument: descriptive and normative content are sharply distinct.
- On the picture I defended, there isn’t a divide between ‘modal’ and ‘nonmodal’ content. The two are merged.
- So there is philosophical work to do to develop, conceptually, a different kind of expressivism.
Thank you!