Indicative Conditionals, Strictly

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Monotonic Patterns

Antecedent Strengthening (AS)

\[ A \rightarrow C \models (A \land B) \rightarrow C \]

Example:

(2)  
   a. If Allie served tea, Chris came.  
   b. So, if Allie served tea and cake, Chris came.

Counterexample (Stalnaker 1968; Adams 1975):

(3)  
   a. If Allie served tea, Chris came.  
   b. # So, if Allie served tea and didn’t invite Chris, Chris came.
Simplification of Disjunctive Antecedents (SDA)

\((A \lor B) \rightarrow C \equiv (A \rightarrow C) \land (B \rightarrow C)\)

Example:

(4)  
   a. If Allie served tea or cake, Chris came.  
   b. So, if Allie served tea, Chris came; and, if Allie served cake, Chris came.

Counterexample (Adams 1975; McKay & van Inwagen 1977):

(5)  
   a. If Allie served only tea or only cake, she served only cake.  
   b. # So, if Allie served only tea, she served only cake.

What to Say...

About Monotonic Patterns?

- Why are they sometimes good and sometimes bad?  
- Current accounts begin with an observation about the felicity of indicative antecedents

Indicative Felicity

An indicative conditional is only felicitous in contexts where its antecedent is mutually supposed to be possible.  
(Stalnaker 1975; Adams 1975; Veltman 1986; Gillies 2010)

(6)  
   a. Allie definitely did not serve tea.  
   b. # If Allie served tea, Chris came.

Antecedent Monotonicity

If \(A \rightarrow C \equiv D\) and \(B \equiv A\), then \(B \rightarrow C \equiv D\)

- Conditional antecedents preserve consequence relations.
- Antecedent Monotonicity follows from Transitivity and the assumption that if \(A \equiv B\) then \(\equiv A \rightarrow B\) (Starr 2019: n22)

Transitivity

\(A \rightarrow B, B \rightarrow C \equiv A \rightarrow C\)

- Antecedent Monotonicity follows from Contraposition and 'Consequent Monotonicity' (Starr 2019: n23)

Contraposition

\(A \rightarrow B \equiv \neg B \rightarrow \neg A\)

Antecedent Strengthening (AS)

\(A \rightarrow C \equiv (A \land B) \rightarrow C\)

Example revisited:

(7)  
   a. Maybe Allie served tea and cake. If Allie served tea, Chris came.  
   b. So, if Allie served tea and cake, Chris came.

Counterexample revisited:

(8)  
   a. Maybe Allie served tea and didn’t invite Chris. # If Allie served tea, Chris came.  
   b. # So, if Allie served tea and didn’t invite Chris, Chris came.
Monotonic Patterns
Returning to the Counterexamples in Light of Indicative Felicity

Simplification of Disjunctive Antecedents (SDA)

\[(A \lor B) \rightarrow C \equiv (A \rightarrow C) \land (B \rightarrow C)\]

Example revisited:

(9) a. Maybe Allie served tea, maybe she served cake. But, if Allie served tea or cake, Chris came.  
   b. So, if Allie served tea, Chris came; and, if Allie served cake, Chris came.

Counterexample revisited:

(10) a. Maybe Allie served only tea. #But, if Allie served only tea or only cake, she served only cake.  
   b. # So, if Allie served only tea, she served only cake.

Two Explanations

Variably-Strict Explanation (Stalnaker 1975)

1. ‘Examples’ are semantically invalid but pragmatically compelling (*reasonable inference*): any context which is updated with a felicitous and true assertion of the premise, is one where the conclusion is true if felicitous.

   2. ‘Counterexamples’ exist because monotonic patterns are semantically invalid, and do not sound pragmatically compelling because Indicative Felicity is not satisfied.

   • **Key Prediction**: any time Indicative Felicity is satisfied, a monotonic pattern will sound compelling.

Strict Explanation

1. ‘Examples’ are compelling because monotonic patterns are semantically valid.

   2. ‘Counterexamples’ sound bad because violation of Indicative Felicity for conclusion leads to:

      • Pragmatical infelicity (Veltman 1986, 1985)  
      • Semantic presupposition failure (Gillies 2004, 2009)  
      • Equivocation via accommodation (Warmbröd 1981)

   • **Key Prediction**: any time Indicative Felicity is satisfied, a monotonic pattern will sound compelling.
Basic Variably-Strict Analysis

A → B is true in a world w, relative to f, just in case all f(A, w)-worlds are B-worlds.

- f(A, w) are the A-worlds most similar to w.
- Context Sensitivity: if w' is in context set c, w' ∈ f(A, w). (Stalnaker 1975)

Basic Strict Analysis

A → B is true in a world w, relative to a space of accessible worlds R(w), just in case all A-worlds in R(w) are B-worlds.

- R(w) the information had by relevant agent’s in w.
- Context Sensitivity: R(w) is the information ‘had’ by the conversationalists in w.
New Counterexamples
SDA and Epistemic Possibility

(11) a. If the coin came up heads or tails, maybe it came up heads.
b. # If the coin came up tails, maybe it came up heads.

(12) a. Maybe the coin came up tails. But, if the coin came up heads or tails, maybe it came up heads.
b. # If the coin came up tails, maybe it came up heads.

- Unlike (10), premise is not infelicitous when conjoined w/conclusion’s presupposition.
- So (11) is a counterexample to the ‘Key Prediction’ of both strict and variably-strict analyses.

New Counterexamples
AS and Epistemic Possibility

(13) a. If Allie served tea, maybe Chris came.
b. # If Allie served tea and Chris didn’t come, maybe Chris came.

(14) a. Maybe Allie served tea and Chris didn’t come. But, if Allie served tea, maybe Chris came.
b. # If Allie served tea and Chris didn’t come, maybe Chris came.

- Unlike (8), premise is not infelicitous when conjoined w/conclusion’s presupposition.
- So (13) is a counterexample to the ‘Key Prediction’ of both strict and variably-strict analyses.

New Counterexamples
AS and Probably

(15) a. If Allie served tea, Chris probably came.
b. # If Allie served tea and Chris didn’t come, Chris probably came.

(16) a. Maybe Allie served tea and Chris didn’t come. But, if Allie served tea, Chris probably came.
b. # If Allie served tea and Chris didn’t come, Chris probably came.

- Premise is not infelicitous when conjoined w/conclusion’s presupposition.
- So (15) is a counterexample to the ‘Key Prediction’ of both strict and variably-strict analyses.
- See Lassiter (2018) for related counterexamples to SDA.
New Counterexamples
Returning to Strict vs. Variably-Strict Debate

- Parallel counterexamples exist for Trans, Contraposition
- This style of counterexample exists for all monotonic patterns
  - While not depending on a violation of Indicative Felicity
- *(Shared)* Key Prediction: any time Indicative Felicity is satisfied, a monotonic pattern will sound compelling.
  - This is false.
- Where should we look for a better analysis?
- Other patterns favor a strict analysis:
  1. Embedded Monotonic Patterns
  2. ‘Preserving Antecedents’ as in Import-Export

Embedded Monotonic Patterns
The Disjunctive Equivalence

(17)  
  a. If Allie served tea, then if Bill brought honey or Chris brought sugar, everyone was happy.  
      \[ A \rightarrow ((B \lor C) \rightarrow H) \]
  b. If Allie served tea, then if Bill brought honey, everyone was happy.  
      \[ A \rightarrow (B \rightarrow H) \]

- Easily predicted by strict analysis via:
  1. The Disjunctive Equivalence
  2. Substitution of equivalent consequents
  3. Consequent weakening
- Not predicted by pragmatic variably-strict analysis:
  - Conditional in consequent of (17) not asserted

Limited Antecedent Weakening
\[ A \rightarrow C, B \rightarrow C \vdash (A \lor B) \rightarrow C \]
- Shared validity in strict/variably-strict analyses

Simplification of Disjunctive Antecedents (SDA)
\[ (A \lor B) \rightarrow C \vdash (A \rightarrow C) \land (B \rightarrow C) \]
- Only valid on strict analysis

The Disjunctive Equivalence
\[ (A \lor B) \rightarrow C \vdash (A \rightarrow C) \land (B \rightarrow C) \]
- Only valid on strict analysis

Embedded cases exist for other monotonic patterns
- E.g. contraposition

Old-style counterexamples lurk here too

(18)  
  a. If Chris came then if Allie served only tea or only cake, she served only cake.  
  b. # If Chris came then if Allie served only tea, she served only cake.

These facts favor strict analyses where Indicative Felicity is treated as a semantic presupposition
- cf. Veltman (1986); Gillies (2009); Stalnaker (1975)
Preserving Antecedents

Antecedent Preservation

**Antecedent Preservation (AP)**

\[ \equiv A \rightarrow (B \rightarrow A) \]

- Valid on strict analysis; not variably-strict analysis.

Example:

(19) If Allie served tea, then if Chris came Allie served tea.

Familiar counterexample:

(20) # If the coin came up heads, then if the coin came up tails it came up heads.

- Both explained on strict analysis w/semantic approach to Indicative Felicity
- No explanation of (19) on variably-strict analysis

Import-Export

**Import-Export**

\[ A \rightarrow (B \rightarrow C) \equiv (A \land B) \rightarrow C \]

- Valid on strict analysis; not variably-strict analysis.

Example:

(22) a. If Allie bet, then if the coin came up heads, she won.
    b. If Allie bet and the coin came up heads, she won.

- No explanation of (22) on variably-strict analysis

Basic Dynamic Semantics

Just Information

**Classical Picture**

- Sentences (relative to contexts) refer to regions of logical space \( W \)
- Interpreters use utterances of them to shift to region of logical space within region referred to

**Dynamic Picture (Veltman 1996; Heim 1982)**

Assign each \( \phi \) a function \( [\phi] \) encoding how it changes \( s \subseteq W \):

\[ s[\phi] = s' \] (i.e.: \( [\phi](s) = s' \))

- Meaning as information update potential.
- \( s \) as mutual information.
The Dynamic Analysis

Conditionals, Epistemic Modals

Dynamic Informational Semantics (Veltman 1996)

Where $s \subseteq W$:

1. $s[A] = \{w \in s \mid w(A) = 1\}$
2. $s[\neg \phi] = s - s[\phi]$
3. $s[\phi \land \psi] = (s[\phi])[\psi]$
4. $s[\phi \lor \psi] = s[\phi] \cup s[\psi]$

Support (Basic Logical Concept)

$s \models \phi \iff s[\phi] = s$

- $s$ supports $\phi$ just in case any information $\phi$ can provide is already part of $s$.

Dynamic Strict Conditional v1 (Gillies 2003, 2009)

$s[\phi \rightarrow \psi] = \begin{cases} s & \text{if } s[\phi] \models \psi \\ \emptyset & \text{otherwise} \end{cases}$

- Tests that all $\phi$-worlds in $s$ are $\psi$-worlds.

Epistemic Modals (Veltman 1996)

1. $s[\Diamond \phi] = \{w \in s \mid s[\phi] \neq \emptyset\}$
2. $s[\Box \phi] = \{w \in s \mid s \models \phi\}$

Strawsonian Dynamic Consequence

$\phi_1, \ldots, \phi_n \models \psi \iff \forall s: \text{if } s[\phi_1] \ldots [\phi_n][\psi] \text{ is defined, then } s[\phi_1] \ldots [\phi_n] \models \psi$

- $s$'s w/failed presuppositions don’t count toward validity (Strawson 1952: 173-9, von Fintel 1999a, Beaver 2001)
- Non-Strawsonian Definition: no conditional validities!
The Dynamic Analysis
And Old Counterexamples: AS

(3) a. If Allie served tea, Chris came.
   b. # So, if Allie served tea and didn’t invite Chris, Chris came.

- $s_0 = \{ w_{AIC}, w_{AIc}, w_{aic} \}$
- Contextually excluded: $w_{AIc}, w_{AIC}, w_{AiC}, w_{aiC}$
- $s_0[A \rightarrow C] = s_0$, since $s_0[A] \models C$.
- But $s_0[A \rightarrow C]$ is undefined.
- So states like $s_0$ don’t count for/against consequence.
- Beauty of Strawsonian Dynamic Consequence at work!

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The Dynamic Analysis
How Does $\Box$ Work?

Persistence (Veltman 1985; Groenendijk et al. 1996)

- $\phi$ is persistent just in case $s' \models \phi$ if $s \models \phi$ and $s' \subseteq s$.
- Support for $\phi$ persists after more information comes in.

- $\Diamond A$ is not persistent.
  - Moving from $s$ to $s'$ can eliminate A-worlds.

Fact (Starr)

If the main consequents are persistent, then antecedent preservation and all monotonic patterns other than contraposition are valid. (Given semantics/logic above.)

Miserly (Veltman 1985)

- $\phi$ is miserly just in case $s' \not\models \phi$ if $s \not\models \phi$ and $s' \subseteq s$.
- $s$ continues to withhold support of $\phi$ even after $s$ is enriched with more information.

- $\Box B$ and $A \rightarrow B$ are not miserly.
  - Moving from $s$ to $s'$ can eliminate $\neg B$-worlds.

Fact (Starr)

If the main consequents are miserly, then contraposition and modus tollens are valid. (Given semantics/logic above.)

(13) a. If Allie served tea, maybe Chris came.
    $A \rightarrow \Diamond C$
   
   b. # If Allie served tea and Chris didn’t come, maybe Chris came.
   $(A \land \neg C) \rightarrow \Diamond C$

- $s_0 = \{ w_{AC}, w_{A\neg C}, w_{aC}, w_{ac} \}$
- $s_0[A \rightarrow \Diamond C] = s_0$, since $s_0[A] \models \Diamond C$
- $s_0[(A \land \neg C) \rightarrow \Diamond C] = \emptyset$, since $s_0[A \land \neg C] \not\models \Diamond C$
- So $s_0[A \rightarrow \Diamond C] \neq (A \land \neg C) \rightarrow \Diamond C$
- Hence: $A \rightarrow \Diamond C \not\models (A \land \neg C) \rightarrow \Diamond C$
- Why? Because of how $\Diamond$ works.

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The Dynamic Analysis

Overall Logic

Unrestricted Validities

1. **Identity**: \( \vdash \phi \rightarrow \phi \)
2. **Modus Ponens**: \( \phi \rightarrow \psi, \phi \vdash \psi \)
3. **Deduction Equivalence**: \( \phi \vdash \psi \iff \vdash \phi \rightarrow \psi \)
4. **Import-Export**: \( \phi \rightarrow (\phi \rightarrow \psi) \vdash (\phi \land \phi) \rightarrow \psi \)

Persistent Validities

For persistent \( \psi \):

1. **Antecedent Strengthening**: \( \phi_1 \rightarrow \psi \vdash (\phi_1 \land \phi_2) \rightarrow \psi \)
2. **SDA**: \( (\phi_1 \lor \phi_2) \rightarrow \psi \vdash (\phi_1 \rightarrow \psi) \land (\phi_2 \rightarrow \psi) \)
3. **Transitivity**: \( \phi_1 \rightarrow \phi_2, \phi_2 \rightarrow \psi \vdash \phi_1 \rightarrow \psi \)
4. **Antecedent Preservation**: \( \vdash \psi \rightarrow (\phi \rightarrow \psi) \)

Miserly Validities

For miserly \( \psi \):

1. **Contraposition**: \( \phi \rightarrow \psi \vdash \neg \psi \rightarrow \neg \phi \)
2. **Modus Tollens**: \( \phi \rightarrow \psi, \neg \psi \vdash \neg \phi \)

- See Veltman (1986) and Yalcin (2012) for MT counterexamples w/non-miserly \( \psi \)

Conditional/Modal Interactions (Gillies 2010)

1. \( \phi \rightarrow \square \psi \models \square (\phi \land \psi) \)
2. \( \square (\phi \rightarrow \psi) \models \square \phi \rightarrow \square \psi \models \square \phi \rightarrow \psi \)

Dynamic Analysis

Review

1. Informational support as basic logical concept
3. Strawsonian Dynamic Logic
   - Modus Ponens, Identity, Import-Export, Deduction Equivalence valid
4. Addresses old-style counterexamples to monotonic patterns and AP
5. New counterexamples explained:
   - AS, SDA, Trans, AP only valid when main consequent is **persistent**
   - CP, MT only valid when main consequent is **miserly**
6. Captures embedded monotonic patterns, AP

Assorted Curiosities

Truth, ‘Probably’ and Subjunctives

\( \{ \mathcal{W} \}, \triangle, < \)
Truth-Conditions Just as Good? (Gillies 2009)

\[ \phi \rightarrow \psi \] 
\[ C = \{ w \mid C(w) \cap [\phi] \subseteq [\psi] \} \]

- All the contextually-live \( \phi \)-worlds are \( \psi \)-worlds
- \( C(w) \) is the set of live worlds with respect to \( w \)
- \( C_\phi(w) = C(w) \cap [\phi] \)

- Modus ponens requires assuming that for all \( w \), \( w \in C(w) \).
- This assumption is inconsistent with interpreting \( C(w) \) as agents’ information.
- That interpretation is essential for basic applications.

Chris just had a brief glimpse at two shapes \( x \) and \( y \).
She thinks there was both a triangle and a square.
Given Chris’ information, is it correct for her to assert/believe:

(23) If \( x \) is a triangle, \( y \) is a square.

- My judgment: Correct.
- As it turns out, \( x \) and \( y \) are both squares.
- Given Chris’ information and the actual state of things, is it correct for her to assert/believe (23)?
  - My judgment: Probably, but some ambivalence.

d’Alembert (1751) on Truth

“The universe... would only be one fact and one great truth for whoever knew how to embrace it from a single point of view.” (d’Alembert 1995: 29)

Truth, Propositions (Starr 2010)

\[ w \models \phi \iff [w][\phi] = \{ w \} \quad [\phi] = \{ w \mid w \models \phi \} \]

Classical Consequence (Starr 2010)

\[ \phi_1, \ldots, \phi_n \models \psi \iff \forall w : [w][\phi_1] \cdots [\phi_n] \models \psi \]

- Classical logic is the logic of perfect information
Truth from a Dynamic Perspective

Truth is Just One Perspective, Man

- Sentences can be evaluated from a range of uncertain (informational) perspectives.
  - That’s a matter of the information supporting the sentence.
- Can also be evaluated from a range of certain (worldly) perspectives.
  - That’s a matter of a world making the sentence true.
- As in (23) after evaluation world is revealed.
- From semantics and truth definition it follows that:

  **Truth-Conditions for Indicative Conditionals**

  1. \( \phi \rightarrow \psi \) is true in \( w \) if \( \phi \land \psi \) is true in \( w \).
  2. \( \phi \rightarrow \psi \) is false in \( w \) if \( \phi \land \neg \psi \) is true in \( w \).
  3. Otherwise, \( \phi \rightarrow \psi \)'s truth-value is undefined.

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Semantics for ‘Probably’
Adapting Yalcin (2012)

\[ s_{Pr}[\Delta \phi] = \begin{cases} s_{Pr} & \text{if } Pr(\{ w \in s : w \models \phi \} \cup \{ w \in s : w \models \phi \lor w \not\models \phi \}) > 0.5 \\ \emptyset_{Pr} & \text{otherwise} \end{cases} \]

- Update clause for atomics must also change to conditionalize \( Pr \); disjunction tricky.

**Interesting Consequences**

1. \( \phi \rightarrow \Delta \psi \models \models \Delta (\phi \rightarrow \psi) \)
2. \( s_{Pr} \models \Delta (A \rightarrow B) \iff Pr([B] | [A]) > 0.5 \)
3. \( \Delta \phi \) is neither persistent nor miserly.

See also de Finetti (1936), Milne (1997), Rothschild (2014).

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Trivalent Truth-Conditions
Why They Matter

- Interaction of quantificational operators and conditionals entails choice:
  1. \( \text{if-clauses are just restrictors (Lewis 1975; Kratzer 1986)} \)
  2. \( \text{Conditionals have trivalent truth-conditions (Jeffrey 1963; Belnap 1970; McDermott 1996; Huitink 2008)} \)
- Option 2 faces serious logical difficulties. Either:
  - \( \neg (\phi \rightarrow \psi) = \phi \) is valid (Huitink 2008; Jeffrey 1963)
  - Modus ponens is invalid (McDermott 1996: 31)
- The account here has no such logical difficulties — logic is not beholden to truth-conditions.
- But it can appeal to those truth-conditions in defining quantificational operators!

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What about Subjunctives?
Antecedent Preservation Failure?

**Antecedent Preservation (AP)**

\( \models A \rightarrow (B \rightarrow A) \)

New indicative counterexample:

(21) If the coin maybe came up heads, then (even) if the coin came up tails, the coin maybe (also) came up heads. \( \diamond H \rightarrow (\neg H \rightarrow \diamond H) \)

- Consider its subjunctive counterpart, in context where we don’t know outcome of past coinflip.

(24) If the coin could have come up heads, then (even) if the coin came up tails, the coin could (also) have come up heads. \( \Diamond \langle \neg H \rangle \rightarrow (\langle \neg H \rangle \rightarrow \Diamond \langle H \rangle) \) (Starr 2014)
What about Subjunctives?

Counterfactual Expansion

**Counterfactual Expansion $\diamond$ (Starr 2014)**

$s_f[\diamond A] = \{ w' \mid \exists w \in s : w' \in f(w, A) \}_f$

- $w'$ is among the A-worlds closest to some $w \in s$
- $w'$ may be outside $s$ (cf. Iatridou 2000; von Fintel 1999b)

- $s_f[\diamond \diamond H \rightarrow (\diamond \neg H \rightarrow \diamond \diamond H)]$ amounts to testing that:
  - $s_f[\diamond \diamond H[\diamond \neg H] = \diamond \diamond H$
  - $s_f[\diamond \diamond H]$ tests that $s_f[\diamond H] \neq \emptyset$.
  - If passed, next step is $s_f[\neg \diamond H]$.
  - This expands to include most-similar $\neg H$-worlds.

- $\diamond \diamond H$ can persist after updating with $\neg H$
- So new counterexamples may not exist for subjunctives...

Allie didn’t host, or serve anything.

(25)  

- a. If Allie had served only tea or only cakes, she could have served only tea.
- b. So, if Allie had served only cakes, she could (also) have served only tea.

- At least much better than indicative counterparts!

### Thank you!

(Slides available at [http://williamstarr.net/research](http://williamstarr.net/research))
References I


References II


References III


References IV


Import-Export

Kaufmann (2005: 213) Counterexample

\[ A \to (B \to C) \models (A \land B) \to C \]

We have a very wet match that is unlikely to light if struck, but will definitely light if thrown in the campfire.

(26)  
   a. If the match lights, it will light if you strike it.
   b. If you strike the match and it lights, it will light.

• (26a) seems false, while (26b) seems logically true.

(27)  
   a. If the match lit, then if it was struck, it lit.
   b. If the match was struck and it lit, then it lit.

• Much less clear that (27a) is false.

• *Tentative conclusion*: original counterexample is due to future tense/discourse relations/word-order.