Anatomy of Arguments in Natural Language Discourse

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A Simple Example

1. Socrates is mortal. Therefore, Socrates is mortal.

$$p \models p$$
A Very Simple Account

- An argument is valid just in case it’s not possible for the premises to be true and the conclusion false.
Of Course, We Have to Control for Ambiguity

2. John is at the bank. Therefore, John is at the bank.
3. I am happy. Therefore, I am happy.
5. Now is now.
More Worrisomely, We Have to Control for Context

3. I am happy. Therefore, I am happy.
5. Now is now.
3. I am happy. Therefore, I am happy.
5. Now is now.
The Traditional Response

- The context must not shift mid-argument.
- An argument is valid just in case for all contexts $c$ and all models $\mathcal{M}$, if the premises are true in $c$ and $\mathcal{M}$, the conclusion is also true in $c$ and $\mathcal{M}$.

See e.g. Kaplan (1989)
Complications

Arguments in Context

Discourse Structure and Argument Individuation

Bonus!

Summing Up
Complications

Arguments in Context

Discourse Structure and Argument Individuation

Bonus!

Summing Up
Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed (6) and (7), with good reason. Yet they did not have any reason to believe (8).

6. If a Republican wins the election, then if the winner is not Reagan, it'll be Anderson.
7. A Republican will win.
8. So, if the winner is not Reagan, it’ll be Anderson.
There was a murder at a mansion. We know that one of the three staff members is the culprit.

9. If the butler is innocent and the gardener is innocent, then the cook is guilty.

10. If the butler is innocent, then if the gardener is innocent, then the cook is guilty.
Trouble

A semantics that validates both MP and Import-Export (assuming classical consequence relation) collapses the conditional into the material conditional (Gibbard, 1981).
Standard Semantics

$$\lbrack if\ p,\ q \rbrack_{c,w} = 1 \text{ iff } \forall w' \in \text{Closest}_c(p, w).\lbrack q \rbrack_{c,w'} = 1.$$

- Standard contextualist semantics (Lewis/Stalnaker) validate MP, but do not explain why (6)–(8) is bad.
- Worse, they invalidate Import-Export, which sounds valid.
Complications

Arguments in Context

Discourse Structure and Argument Individuation

Bonus!

Summing Up
Of Course, How One Resolves Context-sensitivity Matters

- Maybe we just didn’t characterize context-sensitivity correctly?
11. If Jane is out, then she is having fun.
12. Jane is out.
13. So, she (pointing at Mary) is having fun.

15. If the snake escapes it will bite you. If you get the antidote, you will live.

(Stone, 1997; Schlenker, 2013; Bittner, 2014; Brasoveanu, 2010; Stojnić, 2018, 2017)
Basic Idea For Truth-conditions of a Conditional

▶ Standard:
\[
if(q, r) = \{w \mid \forall w' : wRw' \text{ if } w' \in q, w' \in r\}
\]
(Kratzer, 1977, 1981; Kripke, 1963)

▶ Stojnić:
\[
if(p, q, r) = \{w \mid \forall w' : wRw' \text{ if } w' \in p \& w' \in q, w' \in r\}
\]
(Stojnić, 2017, 2018)
Uniformity to Rescue?

- Just as pronouns have to be resolved in a uniform way, so do modals and conditionals.
6. If a Republican wins, then if it is not Reagan, it’ll be Anderson.

7. A Republican will win.

8. So, if it is not Reagan it will be Anderson.
Uniformity is Not the Solution–It’s a Problem

6. If \( i \) a Republican wins, then if \( i \) the winner is not Reagan, it’ll be Anderson.

7. A Republican will win.

8. So, if \( i \) the winner is not Reagan it will be Anderson.

9. If \( i \) the butler didn’t do it and the gardener didn’t do it, then it was the cook.

10. If \( i \) the butler didn’t do it, then if \( i \) the gardener didn’t do it, it was the cook.

Uniformity constraint predicts readings we don’t get.
Uniformity is Not the Solution—It’s a Problem

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8. So, if the winner is not Reagan it will be Anderson.
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▶ Uniformity constraint predicts readings we don’t get.
Uniformity is Not the Solution–It’s a Problem

6. If \( i \) a Republican wins, then if \( i \) the winner is not Reagan, it’ll be Anderson.

7. A Republican will win.

8. So, if \( i \) the winner is not Reagan it will be Anderson.

9. If \( j \) the butler didn’t do it and the gardener didn’t do it, then it was the cook.

10 If \( j \) the butler didn’t do it, then if \( j \) the gardener didn’t do it, it was the cook.

- Uniformity constraint predicts readings we don’t get.
Complications

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Discourse Structure and Argument Individuation

Bonus!

Summing Up
Coherence

Like any discourse, arguments are structured. As in an ordinary discourse, structure builds interpretive connections.

16. Phil tickled Stanley. Liz poked him. (Kehler et al., 2008)

- **Result** $\Rightarrow$ ‘him’ = Phil.
- **Parallel** $\Rightarrow$ ‘him’ = Stanley.

- The problems of identifying coherence relations and resolving semantic ambiguities are mutually constraining.
- Coherence governs the resolution of a pronoun (Stojnić, Stone, and Lepore, 2017, 2013).
Coherence

Like any discourse, arguments are structured. As in an ordinary discourse, structure builds interpretive connections.

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- Result ⇒ ‘him’ = Phil.
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- The problems of identifying coherence relations and resolving semantic ambiguities are mutually constraining.
- Coherence governs the resolution of a pronoun (Stojnić, Stone, and Lepore, 2017, 2013).
Suppose it is raining, and it might not be raining. (Yalcin, 2007)

Coherence relation of Elaboration forces an incoherent reading of (17).
Modus Ponens

6. If a Republican wins the election, then if the winner is not Reagan, it’ll be Anderson.
7. A Republican will win.
8. So, if the winner is not Reagan, it’ll be Anderson.

- Given the effect of *Elaboration* and *Conclusion*, the conclusion expresses a proposition true at a world \( w \), just in case all the (accessible from \( w \)) worlds in which a Republican wins and it is not Reagan, it’s Anderson.
Modus Ponens

6. If $p \land$ a Republican wins the election, then if $q \land$ the winner is not Reagan, it'll be Anderson.
7. $p \land$ a Republican will win.
8. So, if $q \land$ the winner is not Reagan, it is Anderson.
9. If $p \& q$ then the butler is innocent and the gardener is innocent, then the cook is guilty.

10. If $p \& q$ then if $p \& q$ then the gardener is innocent, then the cook is guilty.

- Given the effect of *Elaboration* the two conditionals express the same content.
Arguments and Structure

- Arguments are not merely individuated as sets of premises and conclusions.
- They are partly individuated by their structure.
- The discourse structure in turn builds the underlying informational pattern.
- Capturing the effects of discourse structure allows for a system that provably preserves classical logic.
Context

- Context is a running record of contextual parameters—a Lewisean scoreboard, or a Kaplanean context dynamicized.
- We order parameters by prominence.
- We exploit stack discipline to model this.
Recall the Truth-conditions

$$ \text{if}(p, q, r) = \{ w \mid \forall w' : wRw' \text{ if } w' \in p \& w' \in q, w' \in r \} $$
Modus Ponens

**Conclusion**

\[
\text{If}(\@E, \langle \text{comp} := r \rangle, \\
\quad \text{Elab}(w_0, \text{If}(\@E, \langle \text{comp} := n \rangle, \text{Elab}(w_0, \langle \text{comp} := a \rangle))); \\
\quad \text{Elab}(w_0, \langle \text{comp} := r \rangle)); \\
\text{If}(\@E, \langle \text{comp} := n \rangle, \text{Elab}(w_0, \langle \text{comp} := a \rangle))
\]

- ‘\@E’ denotes the set of top-ranked epistemically accessible worlds (top-ranked possibility) in the current context.
- ‘r’ corresponds to ‘Republican wins’ and ‘n’ to ‘the winner is not Reagan’, and ’a’ to ‘it’s Anderson’.
The Truth-conditions

**Conclusion**

\[\text{If}(\@E, \langle \text{comp} := r \rangle, \quad \text{Elab}(w_0, \text{If}(\@E, \langle \text{comp} := n \rangle, \text{Elab}(w_0, \langle \text{comp} := a \rangle))); \quad \text{Elab}(w_0, \langle \text{comp} := r \rangle)); \quad \text{If}(\@E, \langle \text{comp} := n \rangle, \text{Elab}(w_0, \langle \text{comp} := a \rangle))\]

- If a Republican wins the election, then if the winner is not Reagan, it’ll be Anderson.

\[\{w \mid \forall w' : wRw' \text{ if } w' \in [\@E]^G' \& w' \in n, \text{ then } w' \in a\}\]

\[\quad [\@E]^G' = [\@E]^G \cap r\]

- So if the winner is not Reagan, it is Anderson.

\[\{w \mid \forall w' : wRw' \text{ if } w' \in [\@E]^G'' \& w' \in n, \text{ then } w' \in a\}\]

\[\quad [\@E]^G'' = [\@E]^G \cap r\]
The Truth-conditions

Conclusion

\[
\begin{align*}
\text{If}(\mathcal{E}, \langle \text{comp} := r \rangle, \\
\ \ \ \text{Elab}(w_0, \text{If}(\mathcal{E}, \langle \text{comp} := n \rangle, \text{Elab}(w_0, \langle \text{comp} := a \rangle))); \\
\ \ \ \text{Elab}(w_0, \langle \text{comp} := r \rangle)); \\
\text{If}(\mathcal{E}, \langle \text{comp} := n \rangle, \text{Elab}(w_0, \langle \text{comp} := a \rangle))
\end{align*}
\]

Where \( p \) is the prominent possibility in the original input context:

\[
p \land r \rightarrow ((p \land r \land n) \rightarrow a)
\]

\[
p \land r
\]

\[
\therefore (p \land r \land n) \rightarrow a
\]
9. If the butler is innocent and the gardener is innocent, then the cook is guilty.

10. If the butler is innocent, then if the gardener is innocent, then the cook is guilty.

Conclusion

\[
\text{If}(\text{@}E, \text{And}(\langle \text{comp} := b \rangle)), \text{Elab}(w_0, \langle \text{comp} := g \rangle) \\
\text{Elab}(w_0, \langle \text{comp} := c \rangle); \\
\text{If}(\text{@}E, \langle \text{comp} := b \rangle); \\
\text{Elab}(w_0, \text{If}(\text{@}E, \langle \text{comp} := g \rangle, \text{Elab}(w_0, \langle \text{comp} := c \rangle))
\]

- ‘b’ corresponds to ‘The butler is innocent’, ‘g’ corresponds to ‘The gardener is innocent’, and ‘c’ to ‘The cook is guilty’. 
Conclusion

If (@E, And(⟨comp := b⟩)), Elab(w₀, ⟨comp := g⟩))
   Elab(w₀, ⟨comp := c⟩);
If (@E, ⟨comp := b⟩),
   Elab(w₀, If (@E, ⟨comp := g⟩, Elab(w₀, ⟨comp := c⟩)))

▶ If the butler is innocent and the gardener is innocent, then the cook is guilty.
\{ w | ∀w' : wRw' if w' ∈ [@E]G ∩ b and w' ∈ g then w' ∈ c \}

▶ If the butler is innocent, then if the gardener is innocent, then the cook is guilty.
\{ w | ∀w' : wRw' if w' ∈ [@E]G ∩ b then w' ∈
\{ w'' | ∀w''' : wRw' if w''' ∈ [@E]G ∩ b and w' ∈ g then w''' ∈ c \}\}
Import/Export

Conclusion
If(@E, And(⟨comp := b⟩)), Elab(\(w_0, ⟨comp := g⟩\))
  Elab(\(w_0, ⟨comp := c⟩\));
If(@E, ⟨comp := b⟩,
  Elab(\(w_0, If(@E, ⟨comp := g⟩, Elab(\(w_0, ⟨comp := c⟩))\))
)

Where \(p\) is the prominent possibility in the original input context:

\((p \land b \land g) \rightarrow c\)
\(\therefore p \land b \rightarrow ((p \land b \land g) \rightarrow c)\)
Complications

Arguments in Context

Discourse Structure and Argument Individuation

Bonus!

Summing Up
A Popular Account of Contextual Validity

- Let contexts be Stalnakerian common ground context (CG), and a meaning of a sentence a function encoding its characteristic effect on the CG. (Cf. Stalnaker (1978, 2002).)

- An argument is valid just in case for any $c$, updating $c$ with the premises delivers a context that accepts the conclusion. (E.g., Veltman, 1985; Gillies, 2004 Yalcin, 2012, 2007; Moss, 2015 *inter alia*)
Modus Tollens

There’s an urn with a 100 marbles. 10 are big and blue, 30 big and red, 50 small and blue, and 10 small and red. One marble is randomly selected (you do not know which) (Yalcin, 2012).

18. If the marble is big, then it is likely red.
19. But the marble is not likely red.
20. So, the marble is not big.

Is this a discovery, or a disaster?

Carroll (1894); Kolodny and MacFarlane (2010); Yalcin (2012); Veltman (1985), *inter alia* treat such examples as counterexamples to MT.
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Is this a discovery, or a disaster?

Carroll (1894); Kolodny and MacFarlane (2010); Yalcin (2012); Veltman (1985), *inter alia* treat such examples as counterexamples to MT.
18. If the marble is big, then it’s likely red.
19. But the marble is not likely red.

21. **Contrast**
   
   If(@E, ⟨comp := q⟩, Elab(w₀, Likely(@E, ⟨comp := r⟩))),
   
   Not(Likely(@E, ⟨comp := r⟩))

- ‘@E’ denotes the set of top-ranked epistemically accessible worlds (top-ranked possibility).
- ‘q’ corresponds to ‘the marble is big’ and ‘r’ to ‘the marble is red’.
21. **Contrast**

\[
\text{If}(\@E, \langle \text{comp} := q \rangle, \text{Elab}(w_0, \text{Likely}(\@E, \langle \text{comp} := r \rangle)));
\text{Not}(\text{Likely}(\@E, \langle \text{comp} := r \rangle))
\]

- If the marble is big, then it’s likely red.
  \[
  \{ w \mid \mathcal{P}(\{ w' \mid wRw' \land w' \in \llbracket \@E \rrbracket^{G'} \land w' \in r \}) / \\
  \mathcal{P}(\{ w' \mid wRw' \land w' \in \llbracket \@E \rrbracket^{G'} \}) > .5
  \]
  \[
  \llbracket \@E \rrbracket^{G'} = \llbracket \@E \rrbracket^G \cap q
  \]
- But the marble is not likely red.
  \[
  W \setminus \{ w \mid \mathcal{P}(\{ w' \mid wRw' \land w' \in \llbracket \@E \rrbracket^{G''} \land w' \in r \}) / \\
  \mathcal{P}(\{ w' \mid wRw' \land w' \in \llbracket \@E \rrbracket^{G''} \}) > .5
  \]
  \[
  \llbracket \@E \rrbracket^{G''} = \llbracket \@E \rrbracket^G, \text{ where } G \text{ is the original input context.}
  \]
Modus Tollens

21. **Contrast**(
    If(\(\text{@}E, \langle \text{comp} := q \rangle, \text{Elab}(w_0, \text{Likely}(\text{@}E, \langle \text{comp} := r \rangle)))\);
    Not(Likely(\(\text{@}E, \langle \text{comp} := r \rangle)))
)

- Where \(p\) is the prominent possibility in the original input context (i.e. \(\llbracket \text{@}E \rrbracket^G\)), and \(\triangle(p, q)\) the possibility *that q is likely relative to p*:

\[
q \rightarrow \triangle(p \land q, r) \\
\neg \triangle(p, r) \\
\therefore \neg q
\]
Complications

Arguments in Context

Discourse Structure and Argument Individuation

Bonus!

Summing Up
An argument is more than a set of premises and a conclusion. It’s partly individuated by its structure.

The structure dictates the context-change, which in turn determines the propositional pattern expressed.

This is crucial for individuating argument patterns, and tracking potential equivocation.

Equivocation results not from mere context-shifting, but through a failure to track informational patterns expressed.

Representing the effects of discourse structure in the logical form allows for a system that preserves classical logic.
Thank you!
Bibliography I


Appendix: Operations on Stacks

First I define operations on stacks and sets of stacks, which I will use to define the semantics for our language later on. Formally, a stack is just a function from a finite convex subset of \( \mathbb{N} \) plus \( \text{comp} \) to a set of worlds plus \( \perp \), where ‘\( \perp \)’ denotes an undefined value. (I’ll assume that ‘\( \text{comp} \)’ is a designated position on the stack.
Where the stack is intended to model prominence ranking, ‘\( \text{comp} \)’ is not affecting the prominence ranking.)
Appendix: Operations on Stacks

- Where $m \in \mathbb{N}$, and $i$ is a stack, $i_m$ is the $m^{th}$ member of the stack if $m$ is within the domain of $i$, and $i_m = \perp$ otherwise. ($i_{comp}$ is the member of the stack stored at the designated position $comp$.)— Selecting at a member of the stack.

- Where $G$ is a set of stacks (i.e. a ‘context’), $g$ a stack, and $u$ a world, $G_m = \bigcup_{g \in G} \{u | g_m \neq \perp \& g_m = u\}$, for $m \in \mathbb{N}$ or $m = comp$.—Getting the $m^{th}$ element in the set of stacks $G$.

- For $m, n \in \mathbb{N}$, and a stack $i$, $i_{m,n}$ is a stack $j$ defined on the set $\{0, ..., n - m\} \cup \{comp\}$ such that for $k \in \mathbb{N}, j_k = i_{(m+k)}$ if $j$ is defined on $k$, and $j_{comp} = i_{comp}$.

- Where $G$ is a context, and $g$ and $j$ are stacks, $G_{m,n} = \bigcup_{g \in G} \{j | j = g_{m,n}\}$ and for $H = G_{m,n}, H_{comp} = G_{comp}$.

- For $m \in \mathbb{N}$, and a stack $i$, $i_{m...}$ is the stack $j$ defined on the set $\{k \in \mathbb{N} \mid i \text{ is defined at}(m + k)\} \cup \{comp\}$ such that, for $k \in \mathbb{N}, j_k = i_{(m+k)}$ and $j_{comp} = i_{comp}$.

- Where $G$ is a context, and $g, j$ are stacks, $G_{m...} = \bigcup_{g \in G} \{j | j = g_{m...}\}$ and for $H = G_{m...}, H_{comp} = G_{comp}$. 
Appendix: Operations on Stacks

▶ If $i$ is a stack with a finite domain with maximal element $k - 1$ then for a stack $j$, $i + j$ is a stack $h$ where, for $x \in \mathbb{N}$, $h_x = i_x$ if $i$ is defined at $x$, and $h_x = j(x - k)$ otherwise (and $h_{comp} = i_{comp}$).

▶ Where $u$ is a world and $i$ is a stack, $u, i$ is a stack beginning with $u$, followed by all the members of $i$ in order, and if $u, i = j$, then $j_{comp} = i_{comp}$. — Appending to a stack.

▶ Where $G$ is a context, $u$ is a world, and $g, j$ are stacks, $G_{u...} = \bigcup_{g \in G} \{ j | j = u_a, g \}$ and for $H = G_{u...}$, $H_{comp} = G_{comp}$.

▶ $g[n]g'$ iff $g_m = g'_m$ for $m \neq n$ (where $m, n \in \mathbb{N} \cup \{comp\}$).

▶ $G \sim_n G'$ iff $\{ g' | g[n]g', g \in G \} = \{ g' | g[n]g', g \in G' \}$ (where $n \in \mathbb{N} \cup \{comp\}$).

▶ $G \approx_n G'$ iff $\{ g_0, n + gn+1... | g \in G' \} = G$ and $G_{comp} = G'_{comp}$. 
Appendix: Semantics

The Interpretation of Atoms: The interpretation of an expression $e$, relative to the interpretation function $\mathcal{I}$ a context $G$, and a world $w$:

- $\mathcal{I}(p)$, if $p \in \mathcal{C}$.
  - Constants.

- $G_m$, if $w_m \in \mathcal{V}$ and $m \in \mathbb{N}$
  - Variables.

- $G_{comp}$
  - A designated position on the stack.

- $\emptyset$ if $G_0 = \bot$, $G_0$, if $G_0 \in \mathcal{I}(P)$, and $G_1 \ldots \emptyset$ otherwise.
  - Find the top ranked entity in $G$, satisfying $P$. 
Appendix: Semantics

The Interpretation of Conditions:

- $[\phi = \psi]^{G,w} = D_\omega$, if $[\phi]^{G,w} = [\psi]^{G,w}$; $[\phi = \psi]^{G,w} = \emptyset$, otherwise.
  - Identity.

- $[-\phi]^{G,w} = D_\omega \setminus [\phi]^{G,w}$.
  - Negation.

- $[\phi \land \psi]^{G,w} = [\phi]^{G,w} \cap [\psi]^{G,w}$.
  - Conjunction.
Appendix: Semantics

The Interpretation of Update Expressions

- $[[comp := p]](w, G, H)$ iff $G \sim_{comp} H \& H_{comp} = [p]^{G,w}$
- $[[\phi]](w, G, H)$ if and only if $H = G$ and $w \in [\phi]^{G,w}$
- $[[K; K']](w, G, H)$ iff $\exists G' : [[K]](w, G, G')$ and $[[K']] (w, G', H)$
Appendix: Semantics

The Interpretation of Update Expressions

- $\llbracket \text{MIGHT}(\phi, K) \rrbracket (w, G, H)$ iff there is a $G'$ and $G''$ such that $\llbracket K \rrbracket (w, G, G') \& G' \approx G'' \& G_0'' = G'_{\text{comp}} \cap \llbracket @E \rrbracket^{G,w} \& G'' \sim_{\text{comp}} H \& H_{\text{comp}} = M(\llbracket \phi \rrbracket^{G,w}, G'_{\text{comp}})$

- $\llbracket \text{IF}(\phi, K_1, K_2) \rrbracket (w, G, H)$ iff there is a $G'$, $G''$, $G'''$ and $G''''$ such that $\llbracket K_1 \rrbracket (w, G, G') \& G' \approx G'' \& G'' = G'_{\text{comp}} \cap \llbracket @E \rrbracket^{G,w} \& \llbracket K_2 \rrbracket (w, G'', G''') \& G''' \approx G'''' \& G''''_0 = G'''_{\text{comp}} \cap \llbracket @E \rrbracket^{G'',w} \& G'''' \sim_{\text{comp}} H \& H_{\text{comp}} = \text{Cond}(\llbracket \phi \rrbracket^{G,w}, G'_{\text{comp}}, G'''_{\text{comp}})$
Appendix: Semantics

The Interpretation of Update Expressions

- $\text{AND}(K_1, K_2)(w, G, H)$ iff there is a $G'$, $G''$, $G'''$ and $G''''$ such that $[K_1](w, G, G')$ & $G' \approx G''$ &

  $$G'' = G'_{\text{comp}} \cap [\@E]_{G, w}^G \& [K_2](w, G'', G''') \& G''' \approx G'''' \&$$

  $$G'''' = G'''_{\text{comp}} \cap [\@E]_{G'', w}^G \& G'''' \sim H \&$$

  $$H_{\text{comp}} = G'_{\text{comp}} \cap G''_{\text{comp}}$$

- $\text{NOT}(K)(w, G, H)$ iff there is a $G'$ such that $[K](w, G, G')$ & $G' \sim H$ & $H_{\text{comp}} = [\neg \text{comp}]_{G', w}^G$ &

- $\text{ASSERT}(K)(w, G, H)$ iff there is a $G'$ such that $[K](w, G, G')$ & $G' \approx H$ & $H_0 = G'_{\text{comp}} \cap [\@E]_{G, w}^G \& w \in H_0$
Appendix: Semantics

In order to define the truth-conditions for updates associated with coherence relations, we assume the following abbreviations:

\[ \text{Elab}(\phi, \psi) \text{ iff } \phi \text{ and } \psi \text{ are centered around the same event or entity, i.e. iff the event or scenario described by } \psi \text{ is a part of the scenario described by } \phi. \]

A formula, \( \phi \), is about of body of information \( \theta \) iff, where \( G \) is the input context to \( \phi \), \( \theta = [\@E]^G, w \), where ‘\( E \)’ is a predicate denoting the property of being an epistemically accessible proposition, and thus, ‘\( \@E \)’ denotes the top-ranked epistemically accessible proposition. I use ‘\( \theta_\phi \)’ to denote the body of information that \( \phi \) is about.

\[ \text{Contrast}(\phi, \psi) \text{ iff } \phi \text{ and } \psi \text{ describe contrasting information about some body of information regarding a common topic.} \]
Appendix: Semantics

- \( \text{Elab}(\phi, K) \) \((w, G, H) \) iff there are \( G' \) and \( G'' \) such that
  \[
  G \approx_0 G' \land G'_0 = [\phi]^{G,w} \land [K](w, G', G'') \land G'' \approx_0 H \land H_0 = G''_{comp} \land \text{Elab}(\phi) \]  

- \( \text{Conclusion}(\phi, K) \) \((w, G, H) \) iff there are \( G' \) and \( G'' \) such that
  \[
  G \approx_0 G' \land G'_0 = [\phi]^{G,w} \land [K](w, G', G'') \land G'' \approx_0 H \land H_0 = G''_{comp} \land \text{Conclusion}(\phi) \]  

- \( \text{Contrast}(K_1, K_2) \) \((w, G, H) \) iff there is a \( G' \) and \( G'' \) such that
  \[
  [K_1](w, G, G') \land G' \approx G'' \land G''_0 = [\theta K_1]^{G,w} \land
  [K_2](w, G'', H) \land [\theta K_1]^{G,w} = [\theta K_2]^{G'',w} \land \text{Contrast}(G''_{comp}, H_{comp})
  \]
Appendix: Semantics

Truth, validity, entailment.

- $K$ is true, relative to a context $G$, a world $w$, and a model $\mathcal{M}$, if there is some $H$, s.t. $H \neq \emptyset$ and $\llbracket K \rrbracket(w, G, H)$. $K$ is false (relative to a context $G$, a world $w$, and a model $\mathcal{M}$,) otherwise.

- $K$ is valid iff it’s true in all models.

- $K_1$ entails $K_2$ iff for any model $\mathcal{M}$, any context $G$, and any world $w$ if there is a $G'$ such that $G' \neq \emptyset$ and $\llbracket K_1 \rrbracket(w, G, G')$, then there is a $G''$ such that $G'' \neq \emptyset$ and $\llbracket K_2 \rrbracket(w, G', G'')$. 