Anatomy of Arguments in Natural Language Discourse

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A Simple Example

1. Socrates is mortal. Therefore, Socrates is mortal.

   $p \models p$

A Very Simple Account

- An argument is valid just in case it's not possible for the premises to be true and the conclusion false.

Of Course, We Have to Control for Ambiguity

2. John is at the bank. Therefore, John is at the bank.

More Worrisomely, We Have to Control for Context

3. I am happy. Therefore, I am happy.
4. A: I am happy. B: Therefore, I am happy.
5. Now is now.

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The Traditional Response

- The context must not shift mid-argument.
- An argument is valid just in case for all contexts c and all models $\mathcal{M}$, if the premises are true in c and $\mathcal{M}$, the conclusion is also true in c and $\mathcal{M}$.

See e.g. Kaplan (1989)

Modus Ponens, (McGee, 1985)

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed (6) and (7), with good reason. Yet they did not have any reason to believe (8).

6. If a Republican wins the election, then if the winner is not Reagan, it'll be Anderson.
7. A Republican will win.
8. So, if the winner is not Reagan, it'll be Anderson.

Import-Export

There was a murder at a mansion. We know that one of the three staff members is the culprit.

9. If the butler is innocent and the gardener is innocent, then the cook is guilty.
10. If the butler is innocent, then if the gardener is innocent, then the cook is guilty.

Trouble

- A semantics that validates both MP and Import-Export (assuming classical consequence relation) collapses the conditional into the material conditional (Gibbard, 1981).

Standard Semantics

$$[\![\textit{if } p, q]\!]^{c,w} = 1 \text{ iff } \forall w' \in \textit{Closest}_c(p, w).[\![q]\!]^{c,w'} = 1.$$

- Standard contextualist semantics (Lewis/Stalnaker) validate MP, but do not explain why (6)–(8) is bad.
- Worse, they invalidate Import-Export, which sounds valid.

Of Course, How One Resolves Context-sensitivity Matters

- Maybe we just didn't characterize context-sensitivity correctly?

Of Course, How One Resolves Context-sensitivity Matters

11. If Jane is out, then she is having fun.
12. Jane is out.
13. So, she (pointing at Mary) is having fun.

Modals and Conditionals Are Anaphoric Expressions

14. A wolf might walk in. It would eat you. (Roberts, 1989)
15. If the snake escapes it will bite you. If you get the antidote, you will live.

(Stone, 1997; Schlenker, 2013; Bittner, 2014; Brasoveanu, 2010; Stojnić, 2018, 2017)

Basic Idea For Truth-conditions of a Conditional

- Standard:

  $if(q, r) = \{w \mid \forall w' : wRw' \text{ if } w' \in q,\ w' \in r\}$

  (Kratzer, 1977, 1981; Kripke, 1963)

- Stojnić:

  $if(p, q, r) = \{w \mid \forall w' : wRw' \text{ if } w' \in p \ \&\ w' \in q,\ w' \in r\}$

  (Stojnić, 2017, 2018)

Uniformity to Rescue?

- Just as pronouns have to be resolved in a uniform way, so do modals and conditionals.

Uniformity is Not the Solution–It's a Problem

6. If a Republican wins, then if$_i$ the winner is not Reagan, it'll be Anderson.
7. A Republican will win.
8. So, if$_i$ it is not Reagan it will be Anderson.

Uniformity is Not the Solution–It's a Problem

6. If$_i$ a Republican wins, then if$_i$ the winner is not Reagan, it'll be Anderson.
7. A Republican will win.
8. So, if$_i$ the winner is not Reagan it will be Anderson.
9. If$_i$ the butler didn't do it and the gardener didn't do it, then it was the cook.
10. If$_i$ the butler didn't do it, then if$_i$ the gardener didn't do it, it was the cook.

- Uniformity constraint predicts readings we don't get.

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- Uniformity constraint predicts readings we don't get.

Coherence

Like any discourse, arguments are structured. As in an ordinary discourse, structure builds interpretive connections.

16. Phil tickled Stanley. Liz poked him. (Kehler et al., 2008)

    - *Result* ⇒ ‘him’ = Phil.
    - *Parallel* ⇒ ‘him’= Stanley.

- The problems of identifying coherence relations and resolving semantic ambiguities are mutually constraining.
- Coherence governs the resolution of a pronoun (Stojnić, Stone, and Lepore, 2017, 2013).

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    - *Result* $\Rightarrow$ 'him' $=$ Phil.
    - *Parallel* $\Rightarrow$ 'him'$=$ Stanley.

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- Coherence governs the resolution of a pronoun (Stojnić, Stone, and Lepore, 2017, 2013).

Coherence and Pronoun Resolution

17. Suppose it is raining, and it might not be raining. (Yalcin, 2007)

- Coherence relation of Elaboration forces an incoherent reading of (17).

Modus Ponens

6. If a Republican wins the election, then if the winner is not Reagan, it'll be Anderson.
7. A Republican will win.
8. So, if the winner is not Reagan, it'll be Anderson.

- Given the effect of *Elaboration* and *Conclusion*, the conclusion expresses a proposition true at a world w, just in case all the (accessible from w) worlds in which a Republican wins and it is not Reagan, it's Anderson.

Modus Ponens

6. If $\overbrace{\text{p \& a Republican wins the election}}^{q}$, then if q & the winner is not Reagan, it'll be Anderson.
7. p & a Republican will win.
8. So, if q & the winner is not Reagan, it is Anderson.

Import/Export

9. If p & the butler is innocent and the gardener is innocent, then the cook is guilty.

10. If p & $\overbrace{\text{the butler is innocent}}^{q}$, then if p & q & the gardener is innocent, then the cook is guilty.

- Given the effect of *Elaboration* the two conditionals express the same content.

Arguments and Structure

- Arguments are not merely individuated as sets of premises and conclusions.
- They are partly individuated by their structure.
- The discourse structure in turn builds the underlying informational pattern.
- Capturing the effects of discourse structure allows for a system that provably preserves classical logic.

Context

- Context is a running record of contextual parameters–a Lewisean scoreboard, or a Kaplanean context dynamicized.
- We order parameters by prominence.
- We exploit stack discipline to model this.

Recall the Truth-conditions

$$if(p,\ q,\ r) = \{w \mid \forall w' : wRw' \text{ if } w' \in p \ \&\ w' \in q,\ w' \in r\}$$

Modus Ponens

$$
\begin{array}{l}
\textbf{Conclusion}(\\
\quad \textbf{If}(@E, \langle comp := r \rangle, \\
\qquad\qquad \textbf{Elab}(w_0, \textbf{If}(@E, \langle comp := n \rangle, \textbf{Elab}(w_0, \langle comp := a \rangle))); \\
\quad \textbf{Elab}(w_0, \langle comp := r \rangle))); \\
\quad \textbf{If}(@E, \langle comp := n \rangle, \textbf{Elab}(w_0, \langle comp := a \rangle)) \\
\hfill)
\end{array}
$$

- '$@E$' denotes the set of top-ranked epistemically accessible worlds (top-ranked possibility) in the current context.
- 'r' corresponds to 'Republican wins' and 'n' to 'the winner is not Reagan', and 'a' to 'it's Anderson'.

The Truth-conditions

$$
\begin{array}{l}
\textbf{Conclusion}(\\
\quad \textbf{If}(@E, \langle comp := r \rangle, \\
\qquad\qquad \textbf{Elab}(w_0, \textbf{If}(@E, \langle comp := n \rangle, \textbf{Elab}(w_0, \langle comp := a \rangle))); \\
\quad \textbf{Elab}(w_0, \langle comp := r \rangle)); \\
\quad \textbf{If}(@E, \langle comp := n \rangle, \textbf{Elab}(w_0, \langle comp := a \rangle)) \\
\hfill)
\end{array}
$$

- If a Republican wins the election, then if the winner is not Reagan, it'll be Anderson.

  $\{w \mid \forall w' : wRw' \text{ if } w' \in [\![@E]\!]^{G'} \ \&\ w' \in n, \text{ then } w' \in a\}$

  - $[\![@E]\!]^{G'} = [\![@E]\!]^{G} \cap r$

- So if the winner is not Reagan, it is Anderson.

  $\{w \mid \forall w' : wRw' \text{ if } w' \in [\![@E]\!]^{G''} \ \&\ w' \in n, \text{ then } w' \in a\}$

  - $[\![@E]\!]^{G''} = [\![@E]\!]^{G} \cap r$

The Truth-conditions

$$
\begin{array}{l}
\textbf{Conclusion}(\\
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\quad \textbf{Elab}(w_0, \langle comp := r\rangle)); \\
\quad \textbf{If}(@E, \langle comp := n\rangle, \textbf{Elab}(w_0, \langle comp := a\rangle)) \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad)
\end{array}
$$

- Where p is the prominent possibility in the original input context:

$$
\begin{array}{l}
p \wedge r \rightarrow ((p \wedge r \wedge n) \rightarrow a) \\
p \wedge r \\
\therefore (p \wedge r \wedge n) \rightarrow a
\end{array}
$$

Import/Export

9. If the butler is innocent and the gardener is innocent, then the cook is guilty.
10. If the butler is innocent, then if the gardener is innocent, then the cook is guilty.

$$
\begin{aligned}
&\textbf{Conclusion}(\\
&\ \textbf{If}(@E, \textbf{And}(\langle comp := b \rangle)), \textbf{Elab}(w_0, \langle comp := g \rangle) \\
&\qquad\quad \textbf{Elab}(w_0, \langle comp := c \rangle); \\
&\ \textbf{If}(@E, \langle comp := b \rangle, \\
&\qquad\quad \textbf{Elab}(w_0, \textbf{If}(@E, \langle comp := g \rangle, \textbf{Elab}(w_0, \langle comp := c \rangle)) \\
&\hfill)
\end{aligned}
$$

- ‘b’ corresponds to ‘The butler is innocent’, ‘g’ corresponds to ‘The gardener is innocent’, and ’c’ to ‘The cook is guilty’.

Import/Export

$$\begin{array}{l}
\textbf{Conclusion}(\\
\quad \textbf{If}(@E, \textbf{And}(\langle comp := b\rangle)), \textbf{Elab}(w_0, \langle comp := g\rangle) \\
\qquad\qquad \textbf{Elab}(w_0, \langle comp := c\rangle); \\
\quad \textbf{If}(@E, \langle comp := b\rangle, \\
\qquad\qquad\quad \textbf{Elab}(w_0, \textbf{If}(@E, \langle comp := g\rangle, \textbf{Elab}(w_0, \langle comp := c\rangle)) \\
\hfill)
\end{array}$$

- If the butler is innocent and the gardener is innocent, then the cook is guilty.

  $\{w \mid \forall w' : wRw'$ if $w' \in [\![@E]\!]^G \cap b$ and $w' \in g$ then $w' \in c\}$
- If the butler is innocent, then if the gardener is innocent, then the cook is guilty.

  $\{w \mid \forall w' : wRw'$ if $w' \in [\![@E]\!]^G \cap b$ then $w' \in$ $\{w'' \mid \forall w''' : wRw'$ if $w''' \in [\![@E]\!]^G \cap b$ and $w' \in g$ then $w''' \in c\}\}$

Import/Export

$$
\begin{array}{l}
\textbf{Conclusion}(\\
\quad \textbf{If}({\color{red}@E}, \textbf{And}(\langle comp := b\rangle)), \textbf{Elab}(w_0, \langle comp := g\rangle) \\
\qquad\qquad \textbf{Elab}(w_0, \langle comp := c\rangle); \\
\quad \textbf{If}({\color{red}@E}, \langle comp := b\rangle, \\
\qquad\qquad \textbf{Elab}(w_0, \textbf{If}({\color{red}@E}, \langle comp := g\rangle, \textbf{Elab}(w_0, \langle comp := c\rangle)) \\
\hfill)
\end{array}
$$

- Where p is the prominent possibility in the original input context:

$$
\begin{array}{l}
(p \wedge b \wedge g) \rightarrow c \\
\therefore p \wedge b \rightarrow ((p \wedge b \wedge g) \rightarrow c)
\end{array}
$$

A Popular Account of Contextual Validity

- Let contexts be Stalnakerian common ground context (CG), and a meaning of a sentence a function encoding its characteristic effect on the CG. (Cf. Stalnaker (1978, 2002).)
- An argument is valid just in case for any c, updating c with the premises delivers a context that accepts the conclusion.

(E.g., Veltman, 1985; Gillies, 2004 Yalcin, 2012, 2007; Moss, 2015 *inter alia*)

Modus Tollens

There's an urn with a 100 marbles. 10 are big and blue, 30 big and red, 50 small and blue, and 10 small and red. One marble is randomly selected (you do not know which) (Yalcin, 2012).

18. If the marble is big, then it is likely red.
19. But the marble is not likely red.
20. So, the marble is not big.

Is this a discovery, or a disaster?

Carroll (1894); Kolodny and MacFarlane (2010); Yalcin (2012); Veltman (1985), *inter alia* treat such examples as counterexamples to MT.

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Modus Tollens

18. If the marble is big, then it's likely red.
19. But the marble is not likely red.

21. **Contrast**(
    $\mathbf{If}(@E, \langle comp := q\rangle, \mathbf{Elab}(w_0, \mathbf{Likely}(@E, \langle comp := r\rangle)));$
    $\mathbf{Not}(\mathbf{Likely}(@E, \langle comp := r\rangle))$
)

- '$@E$' denotes the set of top-ranked epistemically accessible worlds (top-ranked possibility).
- 'q' corresponds to 'the marble is big' and 'r' to 'the marble is red'.

Modus Tollens

21. **Contrast**(
    $\textbf{If}(@E, \langle comp := q\rangle, \textbf{Elab}(w_0, \textbf{Likely}(@E, \langle comp := r\rangle)));$
    $\textbf{Not}(\textbf{Likely}(@E, \langle comp := r\rangle))$
)

- If the marble is big, then it's likely red.
  $\{w \mid \mathcal{P}(\{w'|wRw' \;\&\; w' \in [\![@E]\!]^{G'} \;\&\; w' \in r\})/$
  $\mathcal{P}(\{w'|wRw' \;\&\; w' \in [\![@E]\!]^{G'}\}) > .5\}$
  $[\![@E]\!]^{G'} = [\![@E]\!]^{G} \cap q$
- But the marble is not likely red.
  $W \setminus \{w \mid \mathcal{P}(\{w'|wRw' \;\&\; w' \in [\![@E]\!]^{G''} \;\&\; w' \in r\})/$
  $\mathcal{P}(\{w'|wRw' \;\&\; w' \in [\![@E]\!]^{G''}\}) > .5\}$
  $[\![@E]\!]^{G''} = [\![@E]\!]^{G}$, where G is the original input context.

Modus Tollens

21. **Contrast**(
    $\textbf{If}(@E, \langle comp := q\rangle, \textbf{Elab}(w_0, \textbf{Likely}(@E, \langle comp := r\rangle)))$;
    $\textbf{Not}(\textbf{Likely}(@E, \langle comp := r\rangle))$
)

- Where p is the prominent possibility in the original input context (i.e. $[\![@E]\!]^G$), and $\triangle(p, q)$ the possibility *that q is likely relative to p*:

$$q \rightarrow \triangle(p \wedge q, r)$$

$$\neg\triangle(p, r)$$

$$\therefore \neg q$$

Summing Up: I have argued...

- An argument is more than a set of premises and a conclusion. It's partly individuated by its structure.
- The structure dictates the context-change, which in turn determines the propositional pattern expressed.
- This is crucial for individuating argument patterns, and tracking potential equivocation.
- Equivocation results not from mere context-shifting, but through a failure to track informational patterns expressed.
- Representing the effects of discourse structure in the logical form allows for a system that preserves classical logic.

Thank you!

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Appendix: Operations on Stacks

First I define operations on stacks and sets of stacks, which I will use to define the semantics for our language later on. Formally, a stack is just a function from a finite convex subset of $\mathbb{N}$ plus *comp* to a set of worlds plus $\perp$, where '$\perp$' denotes an undefined value. (I'll assume that '*comp*' is a designated position on the stack. Where the stack is intended to model prominence ranking, '*comp*' is not affecting the prominence ranking.)

Appendix: Operations on Stacks

- Where $m \in \mathbb{N}$, and i is a stack, i_m is the m^{th} member of the stack if m is within the domain of i, and $i_m = \perp$ otherwise. (i_{comp} is the member of the stack stored at the designated position *comp*.)— Selecting at a member of the stack.
  - Where G is a set of stacks (i.e. a 'context'), g a stack, and u a world, $G_m = \bigcup_{g \in G}\{u | g_m \neq \perp \ \& \ g_m = u\}$, for $m \in \mathbb{N}$ or $m = comp$.—Getting the m^{th} element in the set of stacks G.

- For m, $n \in \mathbb{N}$, and a stack i, $i_{m,n}$ is a stack j defined on the set $\{0, ..., n - m\} \cup \{comp\}$ such that for $k \in \mathbb{N}, j_k = i_{(m+k)}$ if j is defined on k, and $j_{comp} = i_{comp}$.
  - Where G is a context, and g and j are stacks, $G_{m,n} = \bigcup_{g \in G}\{j | j = g_{m,n}\}$ and for $H = G_{m,n}, H_{comp} = G_{comp}$.

- For $m \in \mathbb{N}$, and a stack i, $i_{m...}$ is the stack j defined on the set $\{k \in \mathbb{N} \mid i \text{ is defined at}(m + k)\} \cup \{comp\}$ such that, for $k \in \mathbb{N}, j_k = i_{(m+k)}$ and $j_{comp} = i_{comp}$.
  - Where G is a context, and g, j are stacks, $G_{m...} = \bigcup_{g \in G}\{j | j = g_{m...}\}$ and for $H = G_{m...}, H_{comp} = G_{comp}$.

Appendix: Operations on Stacks

- If i is a stack with a finite domain with maximal element $k-1$ then for a stack j, $i+j$ is a stack h where, for $x \in \mathbb{N}, h_x = i_x$ if i is defined at x, and $h_x = j_{(x-k)}$ otherwise (and $h_{comp} = i_{comp}$).
- Where u is a world and i is a stack, u, i is a stack beginning with u, followed by all the members of i in order, and if $u, i = j$, then $j_{comp} = i_{comp}$.—Appending to a stack.
  - Where G is a context, u is a world, and g, j are stacks, $G_{u\ldots} = \bigcup_{g \in G}\{j | j = u_a, g\}$ and for $H = G_{u\ldots}$, $H_{comp} = G_{comp}$.
- $g[n]g'$ iff $g_m = g'_m$ for $m \neq n$ (where $m, n \in \mathbb{N} \cup \{comp\}$).
- $G \underset{n}{\sim} G'$ iff $\{g' | g[n]g', g \in G\} = \{g' | g[n]g', g \in G'\}$ (where $n \in \mathbb{N} \cup \{comp\}$).
- $G \underset{n}{\approx} G'$ iff $\{g_{0,n} + g_{n+1\ldots} | g \in G'\} = G$ and $G_{comp} = G'_{comp}$.

Appendix: Semantics

The Interpretation of Atoms: The interpretation of an expression e, relative to the interpretation function $\mathcal{I}$ a context G, and a world w:

- $[\![p]\!]^{G,w} = \mathcal{I}(p)$, if $p \in \mathcal{C}$.
  - Constants.
- $[\![\mathbf{w}_m]\!]^{G,w} = G_m$, if $\mathbf{w}_m \in \mathcal{V}$ and $m \in \mathbb{N}$
  - Variables.
- $[\![comp]\!]^{G,w} = G_{comp}$
  - A designated position on the stack.
- $[\![@P]\!]^{G,w} = \emptyset$ if $G_0 = \bot$, $[\![@P]\!]^{G,w} = G_0$, if $G_0 \in \mathcal{I}(P)$, and $[\![@P]\!]^{G,w} = [\![@P]\!]^{G_{1\ldots},w}$ otherwise.
  - Find the top ranked entity in G, satisfying P.

Appendix: Semantics

The Interpretation of Conditions:

- $[\![\phi = \psi]\!]^{G,w} = \mathcal{D}_\omega$, if $[\![\phi]\!]^{G,w} = [\![\psi]\!]^{G,w}$; $[\![\phi = \psi]\!]^{G,w} = \emptyset$, otherwise.
  - Identity.
- $[\![\neg\phi]\!]^{G,w} = \mathcal{D}_\omega \setminus [\![\phi]\!]^{G,w}$.
  - Negation.
- $[\![\phi \wedge \psi]\!]^{G,w} = [\![\phi]\!]^{G,w} \cap [\![\psi]\!]^{G,w}$.
  - Conjunction.

Appendix: Semantics

The Interpretation of Update Expressions

- $[\![\langle comp := p\rangle]\!](w, G, H)$ iff $G \underset{comp}{\sim} H$ & $H_{comp} = [\![p]\!]^{G,w}$
- $[\![[\phi]]\!](w, G, H)$ if and only if $H = G$ and $w \in [\![\phi]\!]^{G,w}$
- $[\![K; K']\!](w, G, H)$ iff $\exists G' : [\![K]\!](w, G, G')$ and $[\![K']\!](w, G', H)$

Appendix: Semantics

The Interpretation of Update Expressions

- $[\![\text{MIGHT}(\phi, K)]\!](w, G, H)$ iff there is a G' and G'' such that $[\![K]\!](w, G, G')$ & $G' \underset{0}{\approx} G''$ & $G''_0 = G'_{comp} \cap [\![@E]\!]^{G,w}$ & $G'' \underset{comp}{\sim} H$ & $H_{comp} = M([\![\phi]\!]^{G,w}, G'_{comp})$
- $[\![\text{IF}(\phi, K_1, K_2)]\!](w, G, H)$ iff there is a G', G'', G''' and G'''' such that $[\![K_1]\!](w, G, G')$ & $G' \underset{0}{\approx} G''$ & $G''_0 = G'_{comp} \cap [\![@E]\!]^{G,w}$ & $[\![K_2]\!](w, G'', G''')$ & $G''' \underset{0}{\approx} G''''$ & $G''''_0 = G'''_{comp} \cap [\![@E]\!]^{G'',w}$ & $G'''' \underset{comp}{\sim} H$ & $H_{comp} = Cond([\![\phi]\!]^{G,w}, G'_{comp}, G'''_{comp})$

Appendix: Semantics

The Interpretation of Update Expressions

- $[\![\text{AND}(K_1, K_2)]\!](w, G, H)$ iff there is a G', G'', G''' and G'''' such that $[\![K_1]\!](w, G, G')$ & $G' \underset{0}{\approx} G''$ &
  $G''_0 = G'_{comp} \cap [\![@E]\!]^{G,w}$ & $[\![K_2]\!](w, G'', G''')$ & $G''' \underset{0}{\approx} G''''$ &
  $G''''_0 = G'''_{comp} \cap [\![@E]\!]^{G'',w}$ & $G'''' \underset{arg_0}{\sim} H$ &
  $H_{comp} = G'_{comp} \cap G''_{comp}$
- $[\![\text{NOT}(K)]\!](w, G, H)$ iff there is a G' such that $[\![K]\!](w, G, G')$ & $G' \underset{comp}{\sim} H$ & $H_{comp} = [\![\neg comp]\!]^{G',w}$
- $[\![\text{ASSERT}(K)]\!](w, G, H)$ iff there is a G' such that $[\![K]\!](w, G, G')$ & $G' \underset{0}{\approx} H$ & $H_0 = G'_{comp} \cap [\![@E]\!]^{G,w}$ & $w \in H_0$

Appendix: Semantics

In order to define the truth-conditions for updates associated with coherence relations, we assume the following abbreviations:

Elab(ϕ, ψ) iff ϕ and ψ are centered around the same event or entity, i.e. iff the event or scenario described by ψ is a part of the scenario described by ϕ.

A formula, ϕ, is about of body of information θ iff, where G is the input context to ϕ, $\theta = [\![@E]\!]^{G,w}$, where '$E$' is a predicate denoting the property of being an epistemically accessible proposition, and thus, '$@E$' denotes the top-ranked epistemically accessible proposition. I use 'θ_ϕ' to denote the body of information that ϕ is about.

Contrast(ϕ, ψ) iff ϕ and ψ describe contrasting information about some body of information regarding a common topic.

Appendix: Semantics

- $[\![\text{ELAB}(\phi, K)]\!](w, G, H)$ iff there are G' and G'' such that $G \underset{0}{\approx} G'$ & $G'_0 = [\![\phi]\!]^{G,w}$ & $[\![K]\!](w, G', G'')$ & $G'' \underset{0}{\approx} H$ & $H_0 = G''_{comp}$ & $Elab([\![\phi]\!]^{G,w}, H_0)$.
- $[\![\text{CONCLUSION}(\phi, K)]\!](w, G, H)$ iff there are G' and G'' such that $G \underset{0}{\approx} G'$ & $G'_0 = [\![\phi]\!]^{G,w}$ & $[\![K]\!](w, G', G'')$ & $G'' \underset{0}{\approx} H$ & $H_0 = G''_{comp}$ & $Con([\![\phi]\!]^{G,w}, H_0)$.
- $[\![\text{CONTRAST}(K_1, K_2)]\!](w, G, H)$ iff there is a G' and G'' such that $[\![K_1]\!](w, G, G')$ & $G' \underset{0}{\approx} G''$ & $G''_0 = [\![\theta_{K_1}]\!]^{G,w}$ & $[\![K_2]\!](w, G'', H)$ & $[\![\theta_{K_1}]\!]^{G,w} = [\![\theta_{K_2}]\!]^{G'',w}$ & $Contrast(G''_{comp}, H_{comp})$

Appendix: Semantics

Truth, validity, entailment.

- K is true, relative to a context G, a world w, and a model $\mathcal{M}$, if there is some H, s.t. $H \neq \emptyset$ and $[\![K]\!](w, G, H)$. K is false (relative to a context G, a world w, and a model $\mathcal{M}$,) otherwise.
- K is valid iff it's true in all models.
- K_1 entails K_2 iff for any model $\mathcal{M}$, any context G, and any world w if there is a G' such that $G' \neq \emptyset$ and $[\![K_1]\!](w, G, G')$, then there is a G'' such that $G'' \neq \emptyset$ and $[\![K_2]\!](w, G', G'')$.